Letters

Conductor Geometry Independence of Phase Velocity in TEM-Mode Transmission Lines

HARRY E. GREEN

It is well known that the phase velocity of a wave in any two-conductor, TEM-mode transmission line having a homogeneous dielectric filling depends only on the properties of the dielectric and not at all on the conductor geometry. Normally this is demonstrated starting from Maxwell's equations [1]. However it is interesting to recall that transmission line theory can be studied from circuit principles alone without recourse to electrodynamics.

This was the historical order of development. Lord Kelvin in 1855, about a decade in advance of Maxwell's theory, seems to have been the first to study the TEM-mode transmission line in connection with the proposed transatlantic cable. Kelvin, being concerned only with telegraphic signaling, chose to neglect series inductance and shunt leakage and so obtained a one-dimensional diffusion equation for the propagation of current and voltage. Nonetheless, when in 1876 Heaviside gave a complete solution of the problem, he made no use of field theory [2].

The purpose of this note is to show that a proof of the invariance of phase velocity with conductor geometry is possible from essentially circuital considerations. A suitable starting point is Caratheodory's generalization of the Reimann mapping theorem [3]: "a doubly connected domain R whose frontier consists of two continua each containing more than one point can be represented, by a one–one conformal transformation, on an annular region whose frontier consists of two concentric circles."

From this it is obvious that any TEM-mode transmission line which consists of two conductors, each of arbitrary section, one totally enclosed within the other, may be transformed into a concentric, right circular, coaxial line. Since solutions of Laplace's equation are invariant under conformal transformation, the inductance and capacitance per unit length of the original and transformed lines must be the same. To establish that fact generally, it remains then only to observe that for a normal coaxial line, a problem to which there is a simple analytic solution, phase velocity is independent of geometry.

When the two conductors are external to each other and at equal and opposite potentials, as in a two-wire open line, the argument must be modified. It is always possible without effect to enclose the line in a conducting shield at infinity, and passing between the conductors and extending to infinity in each direction there will always be a zero-potential, equipotential surface. Placing a conductor along this equipotential surface also has no effect on the field. It does, however, have the effect of generating a parallel pair of lines in which one conductor now totally encloses the other. Applying the mapping theorem to each in turn then leads to a parallel pair of coaxial lines, to each of which the earlier argument applies.

Since conformal mapping has played such a significant role in the study of particular transmission lines, it is fitting that it also provides a means to demonstrate the geometry independence of phase velocity. From this also follows Willoughby's observation that, for a given homogeneous dielectric filling material, TEM-mode transmission lines having the same characteristic impedance are the conformal transformations of each other [4]. While it is one thing to appreciate this, it is usually quite another to find the transformation.

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Comments on "Numerical Analysis of *H*-Plane Waveguide Junctions by Combination of Finite and Boundary Elements"

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In the above paper,[1] Ise and Koshiba consider a lossy dielectric post in a rectangular waveguide. In figures 6 through 10, they compare their results with results published by us in [1] and infer that their results are more reasonable than ours. We rechecked our results and noted that the results of Ise and Koshiba are indeed the more accurate ones. We found out that the library subroutine that we used in 1985 for calculating Hankel functions with complex arguments was, unfortunately, deficient. This subroutine affected only those results in [1] pertaining to the lossy post case. Since that time the library subroutine had been corrected, and consequently the plots we now obtain using the method of [1] are consistent with those given by Ise and Koshiba in all regions. Computed results for a sample of the cases considered in table I of their paper are tabulated in Table I below. The excellent agreement between the results in the two tables can be easily verified. It was also revealed that luckily the error in the library subroutine had been corrected before the results we presented in our second paper [2] were derived and thus our results in [2] are impeccable.

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[1]K. Ise and M. Koshiba, *IEEE Trans Microwave Theory Tech.*, vol. 36, pp. 1343–1351, Sept. 1988.

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TABLE I
A SAMPLE OF THE CASES CONSIDERED IN TABLE I OF ISE AND KOSHIBA OBTAINED WITH THE METHOD OF REF [1]

ε''	R_a	X_a	R_b	X_b	P
0	0.00000	-3.18724	0.00000	0.00097	1.00000
3	1.70966	-1.47749	0.00099	0 00094	0.75600
11	0.86776	-0.00437	0.00361	0.00061	0.52924
18	0.55413	0.14015	0.00581	0.00003	0.49859
21	0.47830	0.16431	0.00671	0.00030	0.50052
110	0.09762	0.23384	0.01844	0.01678	0.70951
500	0.03692	0.22022	0.01113	0.03451	0.85162
1000	0.02470	0.21174	0.00818	0.03824	0 89303
10000	0.00713	0.19723	0.00275	0 04438	0.96512

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Corrections to "Multiple Dielectric Posts in a Rectangular Waveguide"

CHUNG-I G. HSU AND HESHAM A AUDA

The above paper[1] contains two typographical errors. The minus sign leading the series in equation (39) should be changed to a plus sign, and the "$\tan^{-1}$" term in equation (43) should be multiplied by f_t^0. The results and conclusions of the paper, as well as those of [1], however, are not affected by these errors.

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[1] C.-I G. Hsu and H. A. Auda, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp 883–891, Aug. 1986.

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